Radicals - Square Roots

Objective: Simplify expressions with square roots.

Square roots are the most common type of radical used. A square root "un-squares" a number. For example, because $5^2 = 25$ we say the square root of 25 is 5. The square root of 25 is written as $\sqrt{25}$.

World View Note: The radical sign, when first used was an R with a line through the tail, similar to our perscription symbol today. The R came from the latin, "radix", which can be translated as "source" or "foundation". It wasn't until the 1500s that our current symbol was first used in Germany (but even then it was just a check mark with no bar over the numbers!

The following example gives several square roots:

Example 1.

$\sqrt{1} = 1$	$\sqrt{121} = 11$
$\sqrt{4} = 2$	$\sqrt{625} = 25$
$\sqrt{9} = 3$	$\sqrt{-81} = \text{Undefined}$

The final example, $\sqrt{-81}$ is currently undefined as negatives have no square root. This is because if we square a positive or a negative, the answer will be positive. Thus we can only take square roots of positive numbers. In another lesson we will define a method we can use to work with and evaluate negative square roots, but for now we will simply say they are undefined.

Not all numbers have a nice even square root. For example, if we found $\sqrt{8}$ on our calculator, the answer would be 2.828427124746190097603377448419... and even this number is a rounded approximation of the square root. To be as accurate as possible, we will never use the calculator to find decimal approximations of square roots. Instead we will express roots in simplest radical form. We will do this using a property known as the product rule of radicals

$$\textbf{Product Rule of Square Roots: } \sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$

We can use the product rule to simplify an expression such as $\sqrt{36 \cdot 5}$ by splitting it into two roots, $\sqrt{36} \cdot \sqrt{5}$, and simplifying the first root, $6\sqrt{5}$. The trick in this

process is being able to translate a problem like $\sqrt{180}$ into $\sqrt{36 \cdot 5}$. There are several ways this can be done. The most common and, with a bit of practice, the fastest method, is to find perfect squares that divide evenly into the radicand, or number under the radical. This is shown in the next example.

Example 2.

$$\sqrt{75} \quad 75 \text{ is divisible by } 25, a \text{ perfect square}$$
$$\sqrt{25 \cdot 3} \quad \text{Split into factors}$$
$$\sqrt{25} \cdot \sqrt{3} \quad \text{Product rule, take the square root of } 25$$
$$5\sqrt{3} \quad \text{Our Solution}$$

If there is a coefficient in front of the radical to begin with, the problem merely becomes a big multiplication problem.

Example 3.

$$5\sqrt{63} \quad 63 \text{ is divisible by } 9, a \text{ perfect square}$$
$$5\sqrt{9 \cdot 7} \quad \text{Split into factors}$$
$$5\sqrt{9} \cdot \sqrt{7} \quad \text{Product rule, take the square root of } 9$$
$$5 \cdot 3\sqrt{7} \quad \text{Multiply coefficients}$$
$$15\sqrt{7} \quad \text{Our Solution}$$

As we simplify radicals using this method it is important to be sure our final answer can be simplified no more.

Example 4.

$$\sqrt{72} \quad 72 \text{ is divisible by } 9, a \text{ perfect square}$$
$$\sqrt{9 \cdot 8} \quad \text{Split into factors}$$
$$\sqrt{9} \cdot \sqrt{8} \quad \text{Product rule, take the square root of } 9$$
$$3\sqrt{8} \quad \text{But } 8 \text{ is also divisible by } a \text{ perfect square, } 4$$
$$3\sqrt{4 \cdot 2} \quad \text{Split into factors}$$
$$3\sqrt{4} \cdot \sqrt{2} \quad \text{Product rule, take the square root of } 4$$
$$3 \cdot 2\sqrt{2} \quad \text{Multiply}$$
$$6\sqrt{2} \quad \text{Our Solution.}$$

The previous example could have been done in fewer steps if we had noticed that $72 = 36 \cdot 2$, but often the time it takes to discover the larger perfect square is more than it would take to simplify in several steps.

Variables often are part of the radicand as well. When taking the square roots of variables, we can divide the exponent by 2. For example, $\sqrt{x^8} = x^4$, because we divide the exponent of 8 by 2. This follows from the power of a power rule of expoennts, $(x^4)^2 = x^8$. When squaring, we multiply the exponent by two, so when taking a square root we divide the exponent by 2. This is shown in the following example.

Example 5.

$$
\begin{aligned}
-5\sqrt{18x^4y^6z^{10}} & \quad 18 \text{ is divisible by } 9, a \text{ perfect square} \\
-5\sqrt{9 \cdot 2x^4y^6z^{10}} & \quad \text{Split into factors} \\
-5\sqrt{9} \cdot \sqrt{2} \cdot \sqrt{x^4} \cdot \sqrt{y^6} \cdot \sqrt{z^{10}} & \quad \text{Product rule, simplify roots, divide exponents by } 2 \\
-5 \cdot 3x^2y^3z^5\sqrt{2} & \quad \text{Multiply coefficients} \\
-15x^2y^3z^5\sqrt{2} & \quad \text{Our Solution}
\end{aligned}
$$

We can't always evenly divide the exponent on a variable by 2. Sometimes we have a remainder. If there is a remainder, this means the remainder is left inside the radical, and the whole number part is how many are outside the radical. This is shown in the following example.

Example 6.

$$
\begin{aligned}
\sqrt{20x^5y^9z^6} & \quad 20 \text{ is divisible by } 4, a \text{ perfect square} \\
\sqrt{4 \cdot 5x^5y^9z^6} & \quad \text{Split into factors} \\
\sqrt{4} \cdot \sqrt{5} \cdot \sqrt{x^5} \cdot \sqrt{y^9} \cdot \sqrt{z^6} & \quad \text{Simplify, divide exponents by } 2, \text{remainder is left inside} \\
2x^2y^4z^3\sqrt{5xy} & \quad \text{Our Solution}
\end{aligned}
$$

8.1 Practice - Square Roots

Simplify.

1) $\sqrt{245}$

2) $\sqrt{125}$

3) $\sqrt{36}$

4) $\sqrt{196}$

5) $\sqrt{12}$

6) $\sqrt{72}$

7) $3\sqrt{12}$

8) $5\sqrt{32}$

9) $6\sqrt{128}$

10) $7\sqrt{128}$

11) $-8\sqrt{392}$

12) $-7\sqrt{63}$

13) $\sqrt{192n}$

14) $\sqrt{343b}$

15) $\sqrt{196v^2}$

16) $\sqrt{100n^3}$

17) $\sqrt{252x^2}$

18) $\sqrt{200a^3}$

19) $-\sqrt{100k^4}$

20) $-4\sqrt{175p^4}$

21) $-7\sqrt{64x^4}$

22) $-2\sqrt{128n}$

23) $-5\sqrt{36m}$

24) $8\sqrt{112p^2}$

25) $\sqrt{45x^2y^2}$

26) $\sqrt{72a^3b^4}$

27) $\sqrt{16x^3y^3}$

28) $\sqrt{512a^4b^2}$

29) $\sqrt{320x^4y^4}$

30) $\sqrt{512m^4n^3}$

31) $6\sqrt{80xy^2}$

32) $8\sqrt{98mn}$

33) $5\sqrt{245x^2y^3}$

34) $2\sqrt{72x^2y^2}$

35) $-2\sqrt{180u^3v}$

36) $-5\sqrt{72x^3y^4}$

37) $-8\sqrt{180x^4y^2z^4}$

38) $6\sqrt{50a^4bc^2}$

39) $2\sqrt{80hj^4k}$

40) $-\sqrt{32xy^2z^3}$

41) $-4\sqrt{54mnp^2}$

42) $-8\sqrt{32m^2p^4q}$

8.1

Answers - Square Roots

1) $7\sqrt{5}$

2) $5\sqrt{5}$

3) 6

4) 14

5) $2\sqrt{3}$

6) $6\sqrt{2}$

7) $6\sqrt{3}$

8) $20\sqrt{2}$

9) $48\sqrt{2}$

10) $56\sqrt{2}$

11) $-112\sqrt{2}$

12) $-21\sqrt{7}$

13) $8\sqrt{3n}$

14) $7\sqrt{7b}$

15) $14v$

16) $10n\sqrt{n}$

17) $6x\sqrt{7}$

18) $10a\sqrt{2a}$

19) $-10k^2$

20) $-20p^2\sqrt{7}$

21) $-56x^2$

22) $-16\sqrt{2n}$

23) $-30\sqrt{m}$

24) $32p\sqrt{7}$

25) $3xy\sqrt{5}$

26) $6b^2a\sqrt{2a}$

27) $4xy\sqrt{xy}$

28) $16a^2b\sqrt{2}$

29) $8x^2y^2\sqrt{5}$

30) $16m^2n\sqrt{2n}$

31) $24y\sqrt{5x}$

32) $56\sqrt{2mn}$

33) $35xy\sqrt{5y}$

34) $12xy$

35) $-12u\sqrt{5uv}$

36) $-30y^2x\sqrt{2x}$

37) $-48x^2z^2y\sqrt{5}$

38) $30a^2c\sqrt{2b}$

39) $8j^2\sqrt{5hk}$

40) $-4yz\sqrt{2xz}$

41) $-12p\sqrt{6mn}$

42) $-32p^2m\sqrt{2q}$